BLOW-UP OF SOLUTIONS OF SEMILINEAR PARABOLIC EQUATION WITH NONLINEAR NONLOCAL NEUMANN BOUNDARY CONDITION

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We consider the following nonlocal initial boundary value problem:

$$\begin{cases} u_t = \Delta u + c(x,t)u^p & \text{for } x \in \Omega, \ \ t > 0, \\ \frac{\partial u(x,t)}{\partial \nu} = \int_\Omega k(x,y,t)u^l(y,t)\,dy & \text{for } x \in \partial\Omega, \ \ t > 0, \\ u(x,0) = u_0(x) & \text{for } x \in \Omega, \end{cases} \tag{1}$$

where Ω is a bounded domain in $\mathbb{R}^n$ for $n \geq 1$ with smooth boundary $\partial\Omega$, $p > 0$ and $l > 0$, ν is the unit outward normal. Here $c(x,t)$ is a nonnegative locally Hölder continuous function defined for $x \in \overline{\Omega}$ and $t \geq 0$ and $k(x,y,t)$ is a nonnegative continuous function defined for $x \in \partial\Omega$, $y \in \overline{\Omega}$ and $t \geq 0$. The initial datum $u_0(x)$ is a nonnegative continuously differentiable function in $\overline{\Omega}$.

We prove global existence theorem for $\max(p,l) \leq 1$. Some criteria on this problem which determine whether the solution blows up in a finite time for sufficiently large or for all nontrivial initial data or the solution exists for all time with sufficiently small or with any initial data are also given. Our results depends on the behavior of the coefficients $c(x,t)$ and $k(x,y,t)$ as $t \to \infty$.

The problem (1) with Dirichlet boundary conditions has been considered in [1].

References

1. *Gladkov, A. and Kim, K. I.* Blow-up of solutions for semilinear heat equation with nonlinear nonlocal boundary. *Journal of Mathematical Analysis and Applications* **338** (2008), 264-273.

2. *Gladkov, A. and Kim, K. I.* Uniqueness and nonuniqueness for reaction-diffusion equation with nonlinear nonlocal boundary condition. *Advances in Mathematical Sciences and Applications* **19** (2009), 39-49.